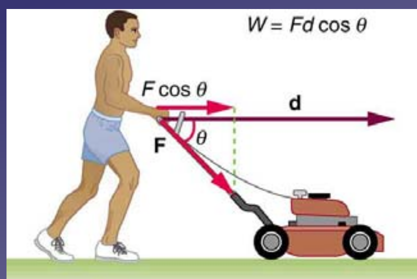
PHYS 210 - General Physics I

- Work



Work

Loosely speaking, work is a measure of "how productive" a force is. It is the energy transferred to/from an object via a force.

The most general definition is:

$$W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r} \qquad \text{where} \qquad d\vec{r} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

- For constant forces,

  where $W = \vec{F} \cdot \Delta\vec{r}$

- Power, P, is the rate at which work is done:

  P = dW/dt

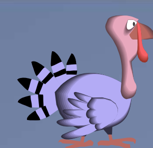
So, the Dot (Scalar) Product ...

The dot (scalar) product is one of two ways to multiply vectors. The result is a scalar.

$$\vec{A} \cdot \vec{B} = AB\cos\theta$$

and

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$



For example...

$$\vec{F} = (2N)\hat{i} + (5N)\hat{j}$$
$$\vec{r} = (3m)\hat{i} - (2m)\hat{j}$$
$$\vec{F} \cdot \vec{r} = ??$$

What is the angle between these two vectors?

$$\vec{F} \cdot \vec{r} = F\, r\cos\theta$$
$$F = \sqrt{(2N)^2 + (5N)^2} = \sqrt{29}$$
$$r = \sqrt{(3m)^2 + (2m)^2} = \sqrt{13}$$
$$\theta = \cos^{-1}\left(\frac{-4Nm}{\sqrt{29}\sqrt{13}Nm}\right) = 102^o$$

Work by Variable Force

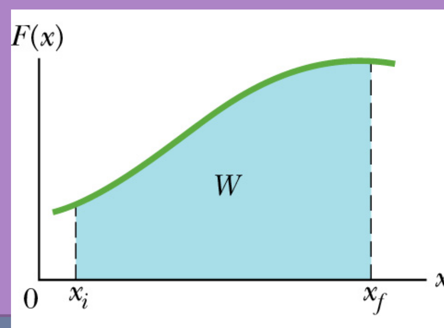
Finally, take the limit that Δx becomes infinitesimally small (dx). We then get a continuous sum, as indicated by the integral sign:

$$W = \int F \cdot dx$$

This gives the area under the curve on an *F vs. x* plot.

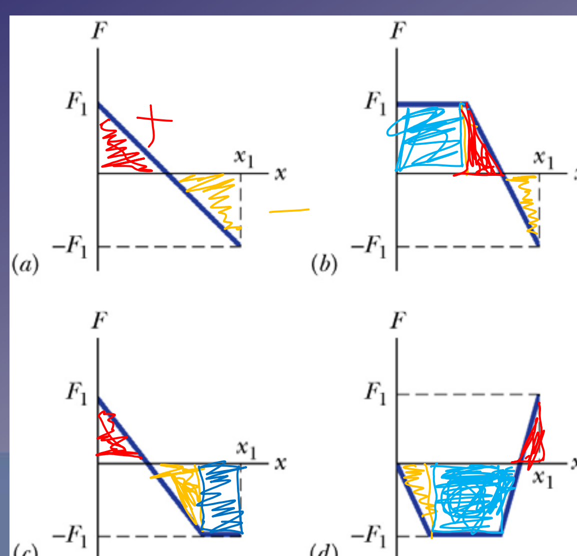
In 3-D, $\boxed{W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}}$

Note the dot product!



Q?

Rank graphs for work done by variable force, F on particle moving from x = 0 to x = x_0, from most positive work to most negative work.



Conservative Forces

- The **Work** done by a conservative force is independent of the path taken; it depends only on the beginning & end points. So, the work done on a closed path is zero.

- Such forces can be written in terms of a potential energy function, **U**, such that

$$\Delta U = -W = -\int_{x_o}^{x} F(x)dx$$

Examples of (mechanical) conservative forces:

- Gravitational forces; gravitational potential energy:

$$U_g(y) = mgy$$

where y is measured relative to an arbitrary reference point.

- Elastic forces; elastic potential energy:

$$U_s(x) = \frac{1}{2}k(\Delta x)^2$$

Where $\Delta x = x_f - x_i$ and x_f is measured relative to the spring equilibrium point, x_i.

Happy Pre-Thanksgiving Weekend!